GENERATION OF THE ELECTROSTATIC FIELD IN THE PULSAR MAGNETOSPHERE PLASMA

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The behaviour of a relativistic electron-positron plasma in the pulsar magnetosphere is investigated. The equation of the motion of the magnetospheric plasma is discussed, from which it follows that, if the plasma particle radial velocity $V_r > c/\sqrt{2}$ (where c is the speed of light), the centrifugal acceleration changes its sign and the particle braking begins. The stability of the magnetospheric plasma with respect to the radially oriented potential perturbations is discussed and the possibility of the electrostatic field generation in this plasma along the pulsar magnetic field lines is shown.

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I. INTRODUCTION

The investigation of the pulsar magnetosphere is of great interest (see, for

example [1-9]). For the determination of the magnetosphere structure it is very important to study the physical processes in plasma of magnetosphere. After the pioneer papers [1,2] it is assumed that, because of the pulsar corotation with its magnetic field, the electric field is generated, which has the nonzero component along the magnetic field. The electric field ejects particles from the pulsar surface and accelerates them up to relativistic velocities. The particles moving along curved magnetic field lines radiate γ-quanta and, when their energy ε_γ exceeds electron's doubled rest energy $2mc^2$ ($\varepsilon_\gamma > 2mc^2$), γ-quantum decays into an electron-positron pair. This pair is also accelerated in the electric field and γ-quanta appear again, which again decay into an electron-positron pairs, etc. This cascade process causes a fast filling of the magnetosphere with the relativistic electron-positron plasma, which, in its turn, screens the electric field generated by the pulsar rotation. In the previous papers (see, for example [4,5]) the strongly turbulized electron-positron plasmas were investigated, but the process of the turbulization was not discussed. We investigate the possible ways of the plasma turbulization, so we discuss the processes in the approximation of the weak turbulence.

In the first section of the present paper the equation of the motion for the magnetospheric plasma in the zeroth approximaton of the weak turbulence is discussed, from which it follows that, if the plasma particle radial velocity $V_r > c/\sqrt{2}$ (where c is the speed of light), the centrifugal acceleration changes its sign and the particle braking begins.

In the second section the equations for the perturbed quantities (in the first approximation of the weak turbulence) are presented. Also the stability of the magnetospheric plasma with respect to the radially oriented potential perturbations is discussed and the possibility of the electrostatic field generation in this plasma along the pulsar magnetic field lines is shown.

II.THE EQUATION OF THE MOTION OF THE MAGNETOSPHERIC PLASMA

The equation of the motion of the magnetospheric plasma particles was discussed in paper [10]. We use the perpendicular rotator model of the pulsar magnetosphere and treat only the polar cap. The magnetic field lines are considered as the radial straight lines located in the plane, which is perpendicular to the pulsar rotation axis. This assumption is justified, because we discuss the processes in the magnetospheric layer, the thickness of which is much less than the curvature radius of the magnetic field lines. The magnetospheric plasma particles move along the pulsar magnetic field lines and also corotate with them, because the field lines are frozen in the plasma. The electric field, generated by the pulsar rotation together with its magnetic field, is screened by the magnetospheric plasma.

It is convenient to begin the discussion of the plasma particle motion in the noninertial frame of a rotating magnetic field line, which is described by the metric:

$$dS^2{=}-(1-\Omega^2r^2)dt^2{+}dr^2, \tag{1}$$

where Ω is the pulsar rotation frequency. Here and below we use so called "geometric units": $c = G = 1$.

According to the Einstein principle of equivalence, we can not tell gravitation from noninertiality. Thus, for the description of the particle motion in the pulsar magnetosphere the "3+1" formalism can be used. This formalism is described in [11]. According to the "3+1" formalism, the equation of the motion for the particle with the mass m and charge e has the following form [11]:

$$\frac{1}{\alpha}\frac{\partial\vec{p}}{\partial t}{+}(\vec{V}\vec{\nabla})\vec{p}{=}-\gamma\frac{\vec{\nabla}\alpha}{\alpha}{+}\frac{e}{m}(\vec{E}+[\vec{V}\vec{B}]), \tag{2}$$

where α is the so called "lapse function" and in our case $\alpha = \sqrt{1-\Omega^2r^2}$. Here and below we use the dimensionless momentum $\vec{p}$ ($\vec{p}$ is changed by $\vec{p}/m$). We can rewrite the equation (2) for the quantities defined in the rest inertial frame:

$$\frac{\partial\vec{p}}{\partial t}{+}(\vec{V}\vec{\nabla})\vec{p}{=}-\gamma\alpha\vec{\nabla}\alpha{+}\frac{e}{m}(\vec{E}+[\vec{V}\vec{B}]). \tag{3}$$

Now let us discuss the motion of the plasma particles in the zeroth approximation of the weak turbulence. In the limits of this approximation the quantities, which are located in the equation of motion, can be presented as:

$$\vec{E}{=}\vec{E_0}{+}\vec{E_1}, \qquad \vec{B}{=}\vec{B_0}{+}\vec{B_1}, \qquad \vec{p}{=}\vec{p_0}{+}\vec{p_1}, \tag{4}$$

where $\vec{E_0}$, $\vec{B_0}$ and $\vec{p_0}$ are the basic terms and $\vec{E_1}$, $\vec{B_1}$ and $\vec{p_1}$ are the perturbations in the first approximation of the expansion over the parameter of the weak turbulence. The small parameter in the approximation of weak turbulence for the electron-positron plasma is:

$$\frac{{E_1}^2}{mn\gamma}{\ll}1. \tag{5}$$

From the equation of the motion in the zeroth approximation for the radial acceleration one can obtain:

$$\frac{\partial p_{0r}}{\partial t}{+}(\vec{V}\vec{\nabla})p_{0r}{=}{-}\gamma_0\Omega^2 r. \tag{6}$$

Let us introduce new variable l, which is connected with coordinate r by the following relation:

$$r = l + \int_{t_0}^{t} v(l,\tau)d\tau. \tag{7}$$

It is clear, that l is a coordinate derived at the moment t_0. Thus, doing the transformation to the Lagrangian variables and using equation (6), we receive (see also [12]):

$$\frac{d^2 l}{dt^2} = \frac{\Omega^2 l}{1-\Omega^2 l^2}\left[1-\Omega^2 l^2 - 2\left(\frac{dl}{dt}\right)^2\right]. \tag{8}$$

Equation (8) can be solved exactly. Using Jacobian functions, the solution can be presented in the form [12]:

$$l(t){=}\frac{V_{0i}}{\Omega}\frac{Sn\Omega t}{dn\Omega t}, \tag{9}$$

where Sn and dn are the Jacobian elliptical sine and modulus respectively [13], V_{0i} is the particle initial velocity. From equation (8) it follows that, if the radial velocity $V_r > 1/\sqrt{2}$, the acceleration changes its sign and the particle is not accelerated, but braked (see also [12]).

In the case $V_{0i}{\rightarrow}1$, using the asymptotic expression for the Jacobian functions one can find [12]:

$$l(t){=}\frac{V_{0i}}{\Omega}sin\Omega t. \tag{10}$$

For the radial velocity we will obtain:

$$V_{0r}{=}V_{0i}cos\Omega t, \tag{11}$$

from which it follows that

$${V_0}^2{=}{(V_{0r})}^2{+}{(V_{0\varphi})}^2{=}const \tag{12}$$

(because of the corotation, $V_{0\varphi} = \Omega r$), i.e. no energy is expending on the particle braking along the field line, the energy transforms from radial to the transversal one.

III. THE EQUATIONS FOR THE PERTURBED QUANTITIES

As it was shown above, the relativistic plasma particles are braked in the pulsar magnetosphere, if their radial velocity $V_r > 1/\sqrt{2}$. It is very interesting to discuss the stability of such a plasma with respect to the radial perturbations. In particular, we discuss the potential perturbations oriented along the magnetic field lines. The initial stage of the perturbation development can be described by the equation, which is easy to obtain from (3) by substituting in it the expansion (4). For the first order terms one can obtain:

$$\frac{\partial\vec{p_1}}{\partial t}{+}(\vec{V_0}\vec{\nabla})\vec{p_1}{=}(\vec{V_0}\vec{p_1})\Omega^2\vec{r}{+}e\vec{E_1}. \tag{13}$$

In order to eliminate the electric field $\vec{E_1}$ from the equation (13), let us use the Poisson equation:

$$div\vec{E_1}{=}4\pi en_1, \tag{14}$$

where $n_1{=}n_{1p}{-}n_{1e}$ is the perturbed concentration — the difference of positron and electron perturbed concentrations.

Let us complete the equations (13) and (14) with the continuity equation in the first approximation of the weak turbulence and thus, lock the set of the equations:

$$\frac{\partial n_1}{\partial t}+divn_0\vec{V_1}+divn_1\vec{V_0}=0. \tag{13}$$

Before solving the set of equations (13)-(15), let us introduce the characteristic spatial parameter of the system and consider the case, when

$$[\frac{\partial v_{0r}}{\partial l}/v_{0r}]^{-1} >> \lambda, \tag{16}$$

where λ is the length of the perturbations. Using the value of v_{0r} obtained from equation (12), equation (16) could be presented in the form:

$$\frac{{R_{LC}}^2 - l^2}{l} >> \lambda, \tag{17}$$

where $R_{LC} = C/\Omega$ is the radius of the Light Cylinder. It follows from equation (17) that, if we investigate the processes in the region with the scale smaller than R_{LC} one, we can consider the medium as homogeneous one and make the Fourier transformation. V_0 - is solved in our description in Lagrangian variables, while the set of equations (13)-(15) is written in the Eulerian variables. So, let us express V_{0r} by the variables r and t (see (7)). Doing the spatial Fourier transformation using approximation (16) for the perturbed quantities P_1, n_1 and E_1, we can obtain equation, which corresponds to the set of the eqs. (13)-(15):

$$\left[\frac{\partial}{\partial t}+ik_rV_{0r}\right]^2 p_{1r}=\frac{\Omega {V_{0i}}^2}{2}\left[\frac{\partial}{\partial t}+ik_rV_{0r}\right]p_{1r}sin2\Omega t-\frac{{\omega_p}^2}{\gamma_0}p_{1r}sin^2 2\Omega t, \tag{18}$$

where ω_p is the plasma frequency.

In order to solve the equation (18), it is convenient to introduce an expression:

$$k_rR(t)=\int k_rV_{0r}(t)dt. \tag{19}$$

Then the equation (18) can be rewritten in the following form:

$$\frac{\partial^2}{\partial t^2}(exp(ik_rR)p_{1_r}(\vec{r},t))=\frac{\Omega {V_{0i}}^2}{2}\frac{\partial}{\partial t}(exp(ik_rR)p_{1_r}(\vec{r},t)sin2\Omega t)-$$
$$-\frac{{\omega_p}^2}{\gamma_0}exp(ik_rR)p_{1_r}(\vec{r},t)sin^2\Omega t. \tag{20}$$

Let us expand the function

$$exp(\pm ik_r R){=}exp(\pm iasin\Omega t), \tag{21}$$

where

$$a{=}\frac{k_r V_0}{\Omega}, \tag{22}$$

in the row over the Bessel functions:

$$exp(\pm iasin\Omega t){=}\sum_{n=-\infty}^{+\infty} I_n(a)exp(\pm in\Omega t). \tag{23}$$

Here $I_n(a)$ is a Bessel function. Doing the expansion (23) in equation (20) and using the Fourier transformation in time, we obtain:

$$\begin{gathered}\frac{{\omega_p}^2}{2\gamma_0}p_{1_r}(\vec{r},\omega){=}\sum_{s,n=-\infty}^{+\infty} I_s(a)I_n(a)(\omega{-}n\Omega)^2 p_{1_r}(\vec{r},\omega{+}(s{-}n)\Omega){-}\\ -\frac{\Omega {V_{0i}}^2}{4}\sum_{s,n=-\infty}^{+\infty} I_s(a)I_n(a)(\omega{-}n\Omega)p_{1_r}(\vec{r},\omega{+}(s{-}n{+}2)\Omega){+}\\ +\frac{\Omega {V_{0i}}^2}{4}\sum_{s,n=-\infty}^{+\infty} I_s(a)I_n(a)(\omega{-}n\Omega)p_{1_r}(\vec{r},\omega{+}(s{-}n{-}2)\Omega){+}\\ +\frac{{\omega_p}^2}{4\gamma_0}p_{1_r}(\vec{r},(\omega{-}2\Omega)){+}\frac{{\omega_p}^2}{4\gamma_0}p_{1_r}(\vec{r},(\omega{+}2\Omega)),\end{gathered} \tag{24}$$

where ω is the frequency of the generated perturbations. We will consider the case of very low frequencies $\omega{\ll}\Omega$, which seems to us to be the most interesting. We will assume that the particle has no time to change its momentum significantly, which fully corresponds to the linear approximation of the weak turbulence and describes the initial stage of the process. In this case all harmonics except the zeroth $p_{1_r}(\vec{r},\omega)$ are small. So keeping only zeroth harmonics in eq.(18), which corresponds to the averaging by the fast oscillations and changing $n{\to}s$, we obtain the dispersion relation:

$$\begin{gathered}\frac{{\omega_p}^2}{2\gamma_0}{=}\sum_{s=-\infty}^{+\infty} {I_s}^2(a)(\omega{-}s\Omega)^2{-}\frac{\Omega {V_0}^2}{4}\Bigg[(\omega{+}2\Omega)\sum_{s=-\infty}^{+\infty} I_s(a)I_{s-2}(a){-}\\ -\Omega\sum_{s=-\infty}^{+\infty} sI_s(a)I_{s-2}(a){-}(\omega{-}2\Omega)\sum_{s=-\infty}^{+\infty} I_s(a)I_{s+2}(a){+}\Omega\sum_{s=-\infty}^{+\infty} sI_s(a)I_{s+2}(a)\Bigg].\end{gathered} \tag{25}$$

It is easy to find that:

$$\sum_{s=-\infty}^{+\infty} s{I_s}^2(a)=0, \qquad \sum_{s=-\infty}^{+\infty} I_s(a)I_{s\pm2}(a)=0, \qquad \sum_{s=-\infty}^{+\infty} sI_s(a)I_{s\pm2}(a)=0,$$

$$\sum_{s=-\infty}^{+\infty} {I_s}^2(a)=1 \quad and \quad \sum_{s=-\infty}^{+\infty} s^2{I_s}^2(a)=\frac{a^2}{2}.$$

So from the (25) we can obtain:

$$\omega^2=\frac{{\omega_p}^2}{2\gamma_0}-\frac{{k_r}^2{V_{0i}}^2}{2}. \tag{26}$$

We know that $E_1\sim exp(-i\omega t)$, so one can conclude that, when the second term in the right hand side of (26) is larger than the first term the aperiodic instability is being developed in the pulsar magnetosphere, i.e. the field E_1 is increasing exponentially along the magnetic field lines.

The condition of the aperiodic instability development can be written in the following form:

$$l_r < \frac{V_{0i}\sqrt{\gamma_0}}{\omega_p}, \tag{27}$$

where l_r is the charge separation scale in the magnetospheric plasma. For the typical parameters of the pulsar magnetosphere the charge separation scale at the light cylinder (the light cylinder is the surface, on which the azimuthal velocity equals to the speed of light $V_\varphi = \Omega r = c$) is of the order $10^6 sm$. As for the parameter $(V_{0i}\sqrt{\gamma_0})/\omega_p$, it differs from the relativistic generalization of the Debye radius l_D [14] ($l_D = (V_T\sqrt{\gamma_T})/\omega_p$, where V_T and γ_T are the particle thermal velocity and Lorentz-factor respectively). On the other hand, it is selfevident that the following condition must be fulfilled:

$$l_r \gg l_D, \tag{28}$$

i.e. the charge separation scale l_r must be much larger than the Debye radius l_D. From the conditions (27) and (28) one can conclude that the aperiodic instability will take place if:

$$\gamma_0 \gg \gamma_{0T}. \tag{29}$$

IV. CONCLUSION

At the end let us discuss the possible results of the instability. We can see that the plasma motion along the magnetic field lines and at the same time rotation together with them (i.e. corotation) cause the generation of the aperiodically increasing electrostatic field under the condition (27). On the other hand, it is selfevident that the corotation can not take place on the arbitrary distances from the pulsar surface, because on some distance the azimuthal velocity will reach the speed of light $V_\varphi = \Omega r = c$. So, the corotation must be removed. The instability, which was discussed above, can contribute to the process of the corotation removing, in particular, the increasing electric field will cause the additional braking of the particles of one sort and the decreasing of braking of the other sort. This fact will evidently cause the motion of the electrons and the positrons with respect to each other, i.e. the increasing current $\vec{j}$ will appeare. So, according to the Maxwell equation $4\pi\vec{j}=rot\vec{B}$, the magnetic field will be generated. The current will be directed along the pulsar magnetic field lines, therefore, the generated magnetic field will have an azimuthal component B_φ. The particles move along the field lines, so, the corotation will be removed. The electric field, i.e. the current $\vec{j}$, will increase untill the corotation law removal.

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